

Modelling of N_2 -thruster plumes based on experiments in STG

Klaus Plähn¹ and Georg Dettleff

Deutsches Zentrum für Luft- Raumfahrt e.V., Bunsenstr. 10, D-37073 Göttingen, Germany

Abstract. In the DLR- high vacuum plume test facility STG experiments have been performed with the aim to validate the available plume flow model. For this purpose fully expanding simulated plumes with N_2 as test gas have been chosen as simplest case (no chemical reactions, constant ratio of specific heats). The essential parameter to be varied was the nozzle flow Reynolds number, the quantities to be measured were the Pitot pressure at the nozzle exit and the molecule number flux in the plume. It turned out that a new model had to be written taking into account in a more appropriate way the development of the boundary layer in the nozzle and the initial plume spreading. The result is a code which describes the plume flow in satisfying agreement with the experimental data.

INTRODUCTION

In the past many attempts have been made to describe the fully expanding thruster plume flow by relatively simple models as well as by sophisticated calculations, but only a few experimental investigations have been performed in the rarefied regime, which is by far the larger and more important part of this flow field with respect to impingement problems [1]. At the last RGD we have reported on molecule number flux measurements in fully expanding free jets and plumes with test gases N_2 and H_2 in the DLR-high vacuum plume test facility STG [2]. These experimental investigations have been continued systematically with the aim to validate the available DLR-model [3] or to improve it. Since there are various parameters influencing an actual thruster plume flow (nozzle shape, chemical reactions and condensation, variation of the ratio of specific heats with decreasing temperature during the expansion, et cetera) it is appropriate to reduce their number at first and to choose the experimental conditions as simple as possible. In this paper we will first present a comparison of the results of such experiments and of calculations with the DLR-model [3]. Since the agreement turned out not to be satisfactory, an analysis of the reasons follows. Based on this a new model is developed and introduced.

EXPERIMENTAL RESULTS

The test facility STG and the free molecule pressure probe to measure the molecule number flux have been introduced previously [4]. In our tests the nozzle shape was conical (DASA 0.5N monopropellant hydrazine thruster nozzle: Half angle 15° , length $l_E = 7.75\text{ mm}$, exit diameter $d_E = 4.75\text{ mm}$, throat diameter $d^* = 0.6\text{ mm}$) and Nitrogen was selected as test gas to have a single component gas flow without interactions and phenomena possibly occurring in a gas mixture. The stagnation temperatures T_0 were 300 K and 600 K ; thus a constant ratio of specific heats κ was maintained throughout the flow. Then the stagnation pressure was varied providing a variation of the nozzle boundary layer thickness $\delta \sim \frac{1}{\sqrt{Re_E}}$ ($Re_E = u_E \rho_E l_E / \mu(T_0)$: nozzle flow Reynolds number; ρ_E , u_E are the velocity and density at the nozzle exit, calculated with the assumption of a pure isentropic flow, and μ is the viscosity). Using a polar coordinate system r, θ (origin at the nozzle exit center, $\theta = 0^\circ$ corresponds to the plume axis), an angular number flux profile family, measured at fixed radius $r = 0.5\text{ m}$ in the range $-90^\circ \leq \theta \leq 90^\circ$ with a variation of Re_E , is shown in **Fig. 1** together with the calculated results for $Re_E = 800$ and 3200 . Different attempts were made to get a better agreement, for example by

¹⁾ now with WABCO Fahrzeugbremsen, Am Lindener Hafen 21, D-30453 Hannover

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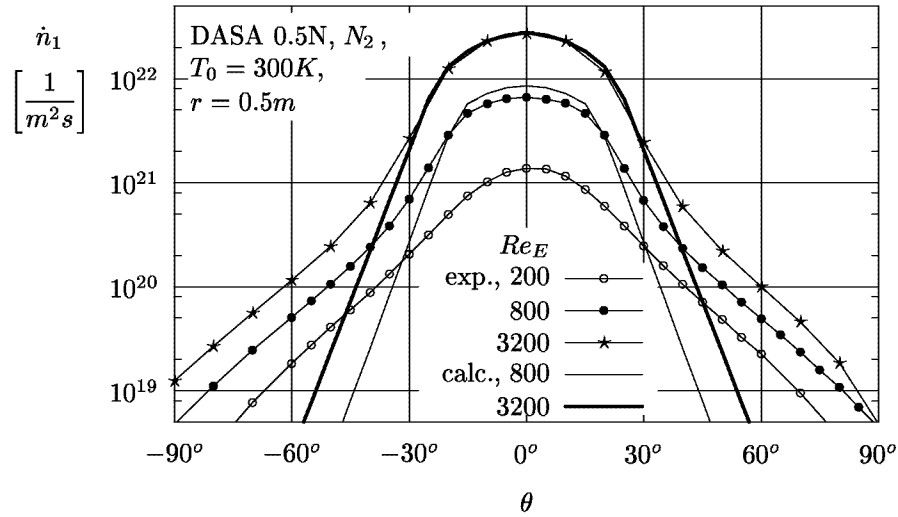


FIGURE 1. Angular profiles of the number flux \dot{n}_1 with different Re_E ; calculations with the former DLR-model

using other simple functions (up to now a \cos -function was used for the isentropic core and an exponential decay to describe the number flux distribution in the boundary layer expansion region). But it turned out that two main features of the profile family in **Fig. 1** could not be reproduced in this way, namely the inflection points in the vicinity of $\theta = \pm 40^\circ$ and the relatively increasing concentration of the number flux in the near axis region with increasing nozzle Reynolds number. Hence it became clear that the proper description of the nozzle boundary layer and of its influence on the isentropic core is decisive and it turned out that the concept used up to now is too simple.

MODELLING

The isentropic core flow in the nozzle was treated as a one-dimensional diverging streamtube which is surrounded by a boundary layer (see **Fig. 2**). The calculation procedure starts in the throat with the assumption of a pure isentropic flow along the length Δx in flow direction. For the next interval Δx downstream the isentropic flow conditions have then to be matched with the boundary layer conditions at the wall in this interval and with the following functions describing the velocity and temperature in the boundary layer, which is assumed to be hypersonic and laminar: Hantzsche and Wendt [5,6] showed in their calculations, that the velocity profile $u(y)$ of the plane hypersonic laminar boundary layer could be approximated satisfactorily by a linear function

$$u(y) = u_\infty \frac{y}{\delta} \quad \text{with} \quad 0 \leq y \leq \delta,$$

where y denotes the distance from the wall, u_∞ the velocity of the flow outside the boundary layer and δ the boundary layer thickness. In [7] Hantzsche and Wendt demonstrate that the boundary layer equations for a flat plate could be transformed into the coordinates of a cone by a simple constant ($C = 0.577$). They demonstrate this for an exterior flow, and we adopt this transformation for the (interior) nozzle flow. The wall temperature T_w , which has been determined by Hantzsche and Wendt [7], Korkegi [8] and van Driest [9] with the assumption of an adiabatic wall, could be approximated by the *recovery* temperature:

$$T_w = T_\infty \left(1 + \sqrt{Pr} \frac{\kappa - 1}{2} Ma_\infty^2 \right)$$

So the temperature profile $T(y)$ could be written as

$$T(y) = (T_w - T_\infty) \cos^{1.5} \left(\frac{\pi y}{2 \delta_T} \right) + T_\infty \quad \text{with} \quad 0 \leq y \leq \delta_T,$$

where δ_T denotes the temperature boundary layer thickness and T_∞ the free stream temperature. The exponent of the \cos -function has been determined empirically by fitting based on Pitot pressure measurements (see **Fig.**

4). In this calculation procedure the nozzle flow Reynolds number is $Re_s = \frac{\rho_w u_{\infty} s}{\mu_w}$ (s denotes the distance along the nozzle wall and $\delta = C \frac{2.85s}{\sqrt{Re_E}}$). With the Prandtl number $Pr = 0.74$ for N_2 the temperature boundary layer thickness δ_T could be estimated with $\frac{\delta}{\delta_T} \approx \sqrt{Pr}$. So the displacement thickness δ_1 of this interval Δx , which reduces the cross-section of the isentropic core in the nozzle, can be calculated with the integration up to δ_T . Matching of isentropic core and boundary layer in every interval is achieved by iteration. The values of the former interval are used as initial values for the iteration of the next interval. So the calculation marches downstream to the nozzle exit (see **Fig. 2**). This concept has also been used by Lengrand [10], but he integrates the *von Kármán* integral equation to determine the displacement thickness.

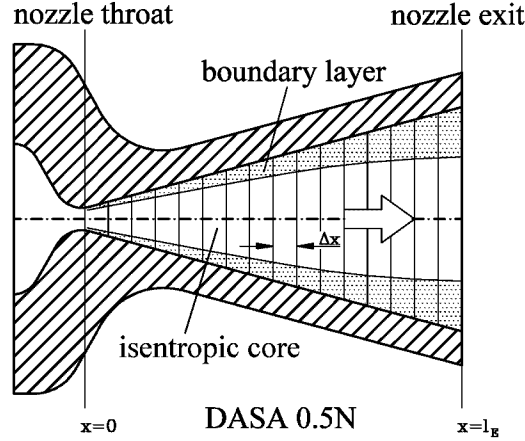


FIGURE 2. Sketch to explain the downstream marching iteration of the nozzle flow

Having calculated the complete nozzle flow in this way (example see **Fig. 3**), Pitot pressure profiles $p_{t2}(y)$ at the nozzle exit were determined both by calculation and measurement and sketched in **Fig. 4** (the details of the measurement, of the concept and of the calculation of the nozzle flow are outlined in [11]). The agreement is considered as satisfactory and the result of this simple approximation of the nozzle flow was therefore considered as suitable to continue with the plume expansion modelling.

In our previous model [3] straight streamlines originated directly at the nozzle exit with an assumed proper assignment of their characterising angle θ with the plume axis to the exit coordinate y perpendicular to the plume axis. Knowing now the number flux distribution at the nozzle exit (from the calculation) and in the plume (from the experiments), it is of course possible to find a better, fitting assignment. However, the former concept means physically that the expansion potential of the gas downstream of the nozzle is neglected (this is clearly to be seen in **Fig. 1**). To consider this expansion potential we tried to apply a concept to the flow

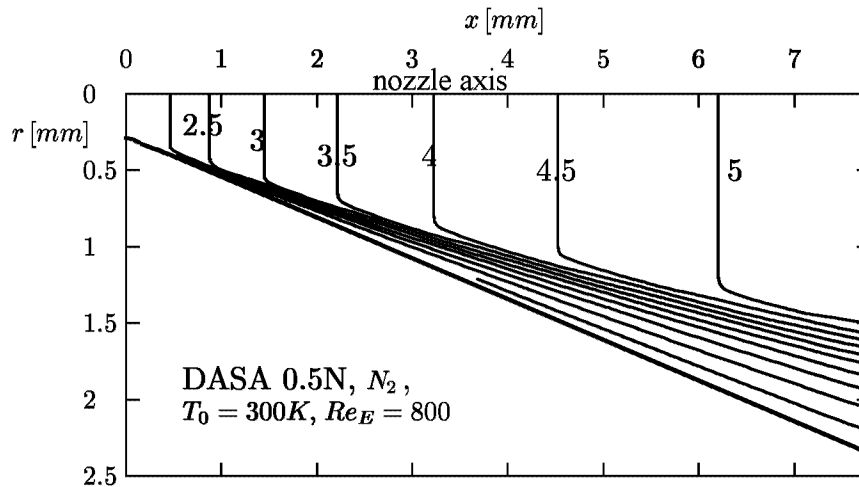


FIGURE 3. Contours of the Mach number in the nozzle

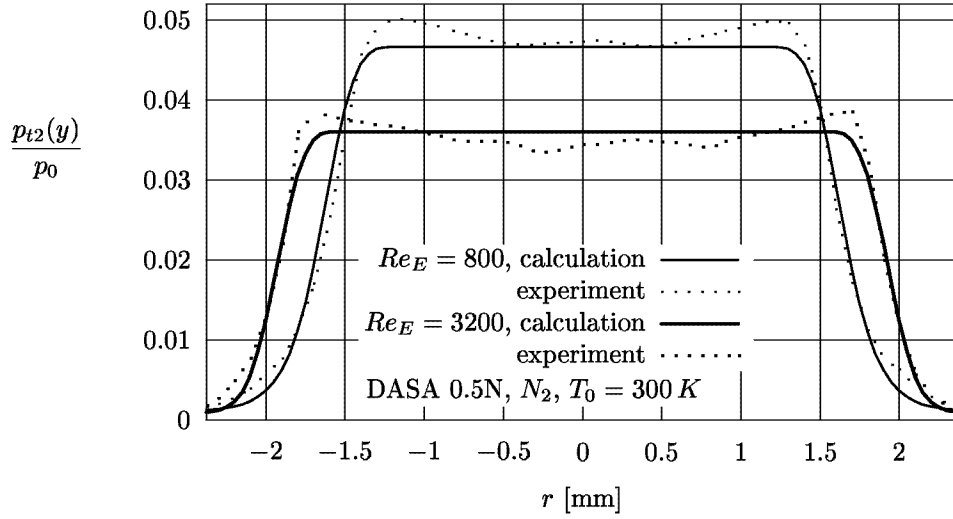


FIGURE 4. Normalized Pitot pressure profiles at the nozzle exit; p_0 : Stagnation pressure

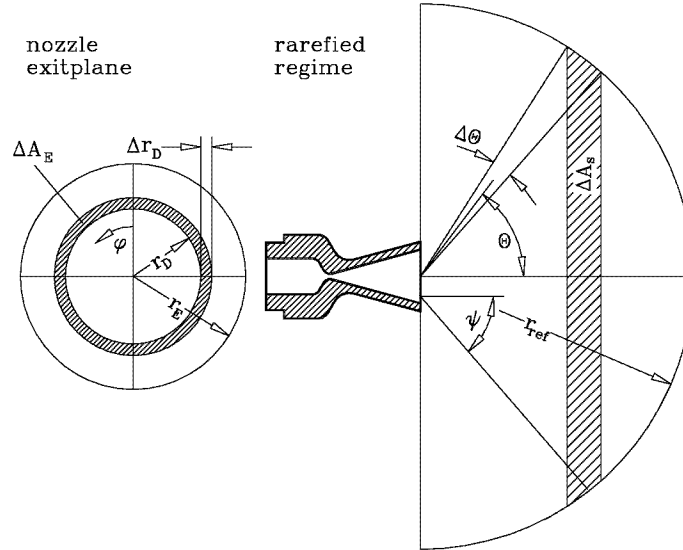


FIGURE 5. Sketch of the surface elements at the nozzle exit ΔA_E and on the reference half sphere ΔA_S

conditions at the nozzle exit, which is as drastic as the one of the freezing surface used to describe the transition to free molecule flow, i.e. a superposition of contributions from different annular surface elements at the nozzle exit (**Fig. 5**). The justification at first is that the flow near the nozzle lip, at least immediately downstream in the off-axis region is rarefied, and the thermal motion of the gas provides a lateral spreading of the molecules with crossing particle paths.

The distribution of the gas portions originating from a surface element ΔA_E (**Fig. 5**) into the different directions ψ is described by an empirical weight function. This weight function is a *Gauß* distribution which is defined as

$$w(\psi) = a e^{-b\psi^2}.$$

For the determination of a and b we first use the mass conservation which is fulfilled by itself (see **Fig. 6**):

$$\int_0^{2\pi} \int_0^{\pi/2} \psi w(\psi) d\psi d\varphi \stackrel{!}{=} 1 \quad \text{and this leads to} \quad a = \left[\frac{\pi}{b} \left(1 - e^{-b\pi^2/4} \right) \right]^{-1}.$$

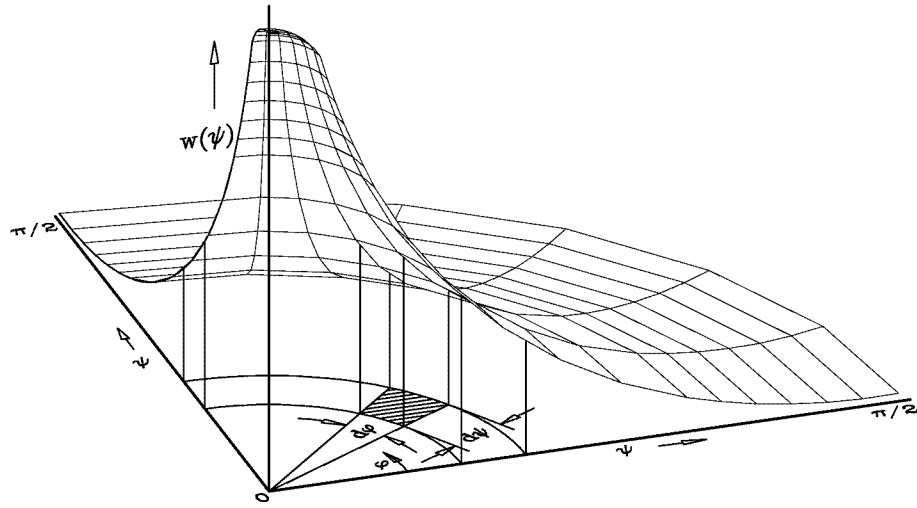


FIGURE 6. Weight function $w(\psi)$ to demonstrate the geometric relations

The quantity b is set to $b = (\sqrt{\kappa/2} Ma)^\kappa$. It depends on the ratio of specific heats κ and the Mach number Ma , since they are the most important parameters which determine the expansion potential at a certain surface element ΔA_E at the nozzle exit. The resulting function $w(\psi)$ for $\kappa = 1.4$ with Ma as parameter is shown in **Fig. 7**. This particular introduction of b was empirically: In the isentropic core it was found that with the calculated Mach numbers at the nozzle exit ($Ma_{E, is} \approx 5.5$) most of the mass flux is spread into a region of $0 \leq \theta \leq 30^\circ$ (see also **Fig. 1**), and if we consider the continuum near axis flow at the nozzle exit as single stream tube, the function $w(\psi)$ applied to this flow yields essentially $\dot{n}_1(\theta)$ as it is found in **Fig. 1** in the plume near axis region. The application to surface elements ΔA_E delivers a refinement but no basic change. This is the reason why we can apply this concept with the weight function to the whole nozzle exit flow.

To simplify the calculation the difference between θ and ψ (**Fig. 5**) for the same surface element ΔA_s is neglected: $\theta = \psi$, and the number of the surface elements at the nozzle exit and at the half sphere r_{ref} is set to k , which leads to

$$\begin{aligned} \Delta r_D &= \frac{r_E}{k}, & r_{D,i} &= i \Delta r_D, & \text{for } i &= 0, \dots, k-1 \\ \Delta \theta &= \frac{\pi}{2k}, & \theta_j &= (j - \frac{1}{2}) \Delta \theta, & \text{for } j &= 1, \dots, k. \end{aligned}$$

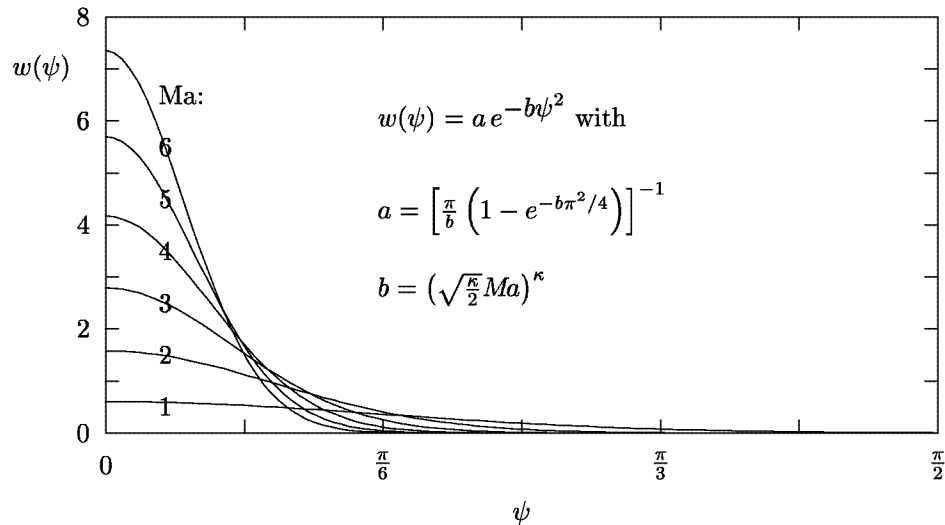


FIGURE 7. Weight function $w(\psi)$ for different Mach numbers and $\kappa = 1.4$

Hence the number flux $\dot{n}_1(r_{ref}, \theta_j)$ through a surface element

$$\Delta A_{s,j} = 2\pi r_{ref}^2 \left[\cos\left(\theta_j - \frac{\Delta\theta}{2}\right) - \cos\left(\theta_j + \frac{\Delta\theta}{2}\right) \right]$$

on a half sphere at the distance r_{ref} can be calculated as the sum of all weighed number fluxes from the surface elements

$$\Delta A_{E,i} = \pi (\Delta r_D^2 + 2r_{D,i} \Delta r_D)$$

at the nozzle exit :

$$\dot{n}_{1,j}(r_{ref}, \theta_j) = \frac{1}{\Delta A_{s,j}} \sum_{i=0}^{k-1} w_{E,ij} u(y_i) \rho(y_i) \frac{\Delta A_{E,i}}{m} \quad , \text{ with } y_i = r_E - \left(r_{D,i} + \frac{\Delta r_D}{2} \right) .$$

The values of the weight function are given by

$$w_{E,ij} = 2\pi a_i \int_{\theta_j - \frac{\Delta\theta}{2}}^{\theta_j + \frac{\Delta\theta}{2}} \psi e^{-b_i \psi^2} d\psi \quad , \text{ with } a_i, b_i \text{ parameters of } w(\psi) \text{ depending on } Ma(y_i) .$$

To avoid freezing on the outer surface elements ΔA_s during the superposition from the nozzle exit plane to the half sphere r_{ref} , the calculation of the number flux is repeated with decreasing radius r_{ref} until the flow on no outer surface element is frozen. Starting at this first half sphere r_{ref} , the calculation of the whole plume marches downstream with an assumed density decay $\rho \sim r^{-2}$ like in the former model (example see **Fig. 8**; x, y denote in this and the next figure cartesian coordinates with the origine in the nozzle exit). In this region freezing is taken into consideration by the calculation of a *freezing* surface [3] at $P_f = 2$ (*Bird's freezing* parameter; see **Fig. 9**).

Two results of the present calculation in comparison to the experiments are shown in **Fig. 10** with the experimental curves taken from **Fig. 1**. The Reynolds numbers represent two extreme cases of boundary layer

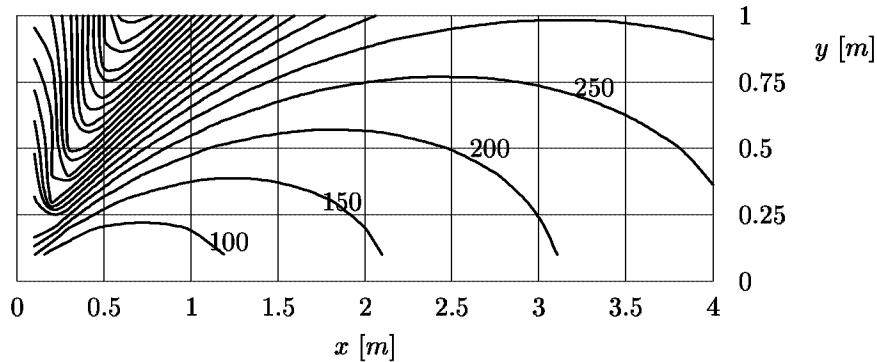


FIGURE 8. Contours of the calculated Mach numbers Ma in the rarefied regime

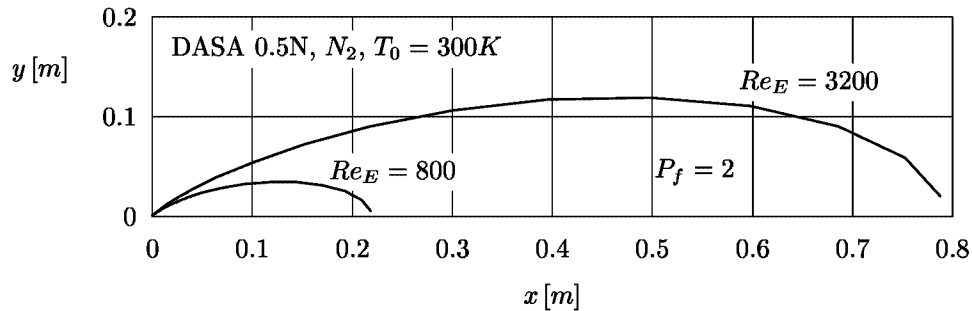


FIGURE 9. Location of the calculated *freezing* surface with different Re_E

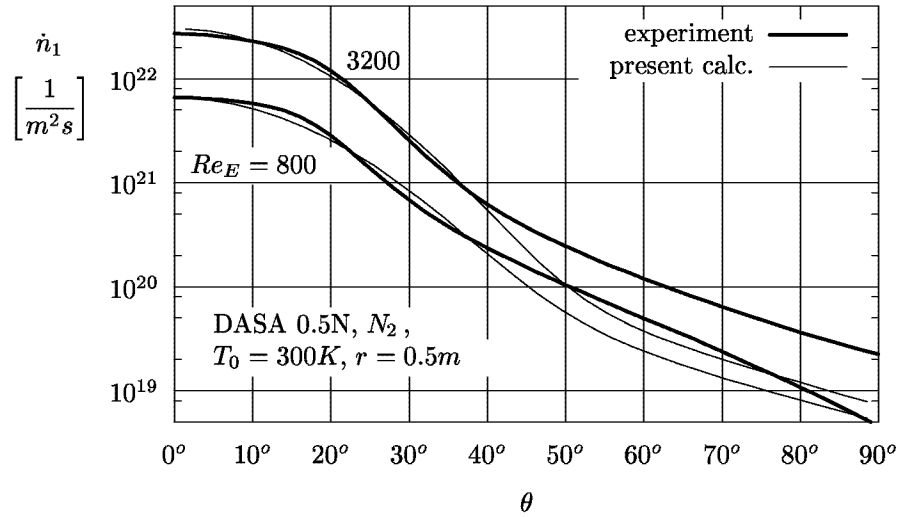


FIGURE 10. Results of the present calculation in comparison to the experiments

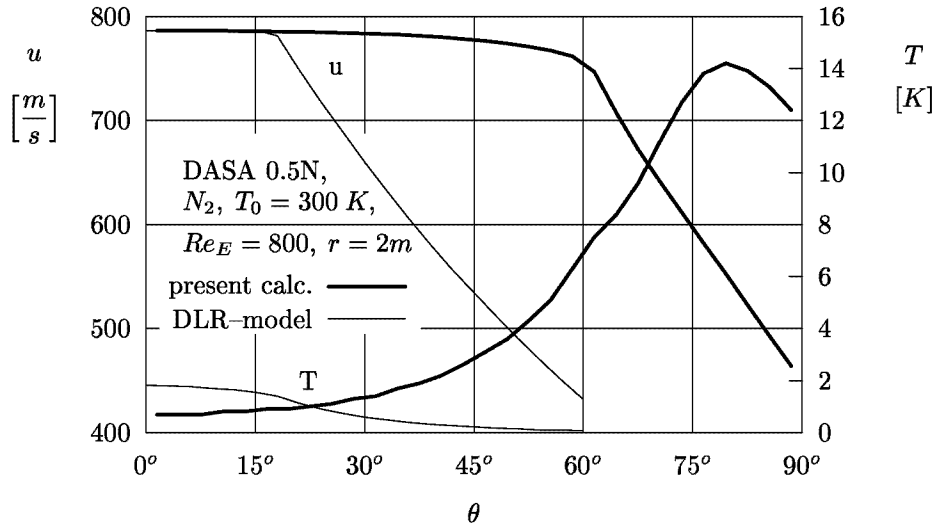


FIGURE 11. Comparison of the angular velocity and temperature distributions: Present calculation and former DLR-model

thickness regarding the actual (monopropellant hydrazine) thruster operation. Qualitatively the curves have the same features, especially the inflection points at $\theta \approx 40^\circ$ are reproduced. The agreement on the axis is very good, in the off-axis region the discrepancy has been reduced drastically compared to that of our previous model (see Fig. 1).

Just for completeness we want to mention that in addition to the number flux other flow quantities like the velocity u and the temperature T can be calculated. During the superposition of the weighted contributions the other flow quantities are weighted by their kinetic energy, and they are calculated by the method of effective stagnation conditions (see [3]). A direct comparison with experimental results is not yet possible, but from the physical plausibility it seems that the results obtained with the new model are more realistic (see Fig. 11).

CONCLUSIONS

The aim of our study was to validate or to improve the DLR- plume model based on experiments with simulated plumes in STG. The measurements especially in the boundary layer expansion region showed that the available model concept was not sufficient, but that the nozzle flow boundary layer characteristics and the initial turning of the streamlines during the free expansion had to be taken stronger into the considerations. We used available boundary layer calculation concepts and applied them to the nozzle flow. By means of Pitot pressure measurements at the nozzle exit location the reliability of the nozzle flow calculations could be shown. A weight function which is essentially determined by Ma and κ was then introduced to perform the transition from the flow at the nozzle exit to a reference half sphere downstream. Thus the initial turning of the streamlines in the free expansion was modelled. The flow quantities at this reference surface are the starting condition for the calculation of the whole plume. We found that the overall resulting number flux distribution in the plume shows a satisfying agreement with the measured one, especially when we regard that the magnitude of this flow quantity changes within this local distribution considerably (relative change from about 10^3 at $\theta = 0^\circ$ to 1 at $\theta = 90^\circ$). A simple plume description was maintained (the duration of the present calculation on a normal personal computer is about one minute to estimate the nozzle flow and the rarefied regime up to a distance of $r = 5m$), and it is intended to further develop the concept to more complicated cases, at first using H_2 , where κ is not constant, and test gas mixtures with N_2 and H_2 to simulate more realistic actual thruster plume conditions.

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